RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JUNE 2022

THIRD YEAR (BATCH 2019-22)

MATHEMATICS (HONOURS)

Date : 13/06/2022

Time : 11.00 am – 1.00 pm

Paper : DSE-3 [Calculus of Variation & Classical Mechanics]

Full Marks : 50

[All the symbols have their usual meaning]

Group – A

Answer **all** the questions, maximum one can score 25.

1. Let $\bar{x}(t)$ be the curve which minimizes the functional $I[x(t)] = \int_0^1 (x^2 + \dot{x}^2)\,dx \left(where\, \dot{x} \equiv \frac{dx}{dt} \right)$, satisfying $x(0)=0, x(1)=1$. Then find $\bar{x}(0,5)$. [6]
2. Find the extremum of the functional $I[y(x)] = \int_0^1 \left((y')^2 + 2y^2 \right) dx.\ y(0)=0, y(1)=1.$ Using Ritz method (for n =2). [6]
3. Show that Jacobi condition for the central field of extremals for

$$I[y(x)] = \int_0^{\frac{\pi}{2}} \left(xyy' + y^2 - \frac{(y')^2}{2} \right) dx$$

is satisfied. Further show that for extremals the functional has weak minima. [6]

4. Obtain Euler-Ostrogradsky equation for

$I[u(x,y,z)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \right] dxdydz$: where the values of *u* are prescribed on the boundary C of the domain *D*. [4]

5. Define Central field. [2]
6. Find the extremal of the functional $I[y(x), z(x)] = \int_0^1 \left(4 + (y')^2 + (z')^2 \right) dx$ that satisfy the boundary conditions $y(0)=0, y(1)=2, z(0)=0$ and $z(1)=4$. [6]

Group – B

Answer **all** the questions, maximum one can score 25.

7. Given $L = \frac{e^t}{2}\left(\dot{x}^2 - k^2x^2\right)$, where *k* is a constant. Show that $\ddot{x} + \dot{x} + k^2x = 0.$ [4]
8. Given the Lagrangian $L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$, find the Hamiltonian and hence Hamilton's canonical equations. [6]
9. A particle of unit mass moves in the direction of x-axis such that it has the Lagrangian $L = \frac{\dot{x}^4}{l2} + \frac{x\dot{x}^2}{2} - x^2$. Let $Q = \dot{x}^2\ddot{x}$ represents a force (not arising from a potential) acting on the particle in the x-direction. If $x = 1 = \dot{x}$ when $t = 0$ then find $\dot{x}$ when $x = 0.5$. [6]

10. In a dynamical system the kinetic energy and potential energy are given by

$$T = \frac{1}{2}\left(\frac{\dot{q}_1^2}{a + bq_2^2} + \dot{q}_2^2\right), V = c + dq_2^2$$

where $a, b, c, d > 0$. Determine $q_1(t)$ and $q_2(t)$ using Routhian. [7]

11. Deduce Lagrange's equation of motion (1st kind) for holonomic and non-holonomic cases. [7]

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